44’th IMO 2003

A1. S is the set {1, 2, 3, ... , 1000000}. Show that for any subset A of S with 101 elements we can find 100 distinct elements xi of S, such that the sets xi + A are all pairwise disjoint. [Note that xi + A is the set {a + xi | a is in A}].

A2. Find all pairs (m, n) of positive integers such that m2/(2mn2 - n3 + 1) is a positive integer.

A3. A convex hexagon has the property that for any pair of opposite sides the distance between their midpoints is (v3)/2 times the sum of their lengths. Show that all the hexagon's angles are equal.

B1. ABCD is cyclic. The feet of the perpendicular from D to the lines AB, BC, CA are P, Q, R respectively. Show that the angle bisectors of ABC and CDA meet on the line AC iff RP = RQ.

B2. Given n > 2 and reals x1 = x2 = ... = xn, show that (?i,j |xi - xj|)2 = (2/3) (n2 - 1) ?i,j (xi - xj)2. Show that we have equality iff the sequence is an arithmetic progression.

B3. Show that for each prime p, there exists a prime q such that np - p is not divisible by q for any positive integer n.